# Beam Pattern Optimization Via Unequal Ascending Clusters

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Abstract - In this paper, two different architectures based on completely and sectionally clustered arrays are proposed to improve the array patterns. In the wholly clustered arrays, all elements of the ordinary array are divided into multiple unequal ascending clusters. In the sectionally clustered arrays, two types of architectures are proposed by dividing a part of the array into clusters based on the position of specific elements. In the first architecture of sectionally clustered arrays, only those elements that are located on the sides of the array are grouped into unequal ascending clusters, and other elements located in the center are left as individual and unoptimized items (i.e. uniform excitation). In the second architecture, only some of the elements close the center are grouped into unequal ascending clusters, and the side elements were left individually and without optimization. The research proves that the sectionally clustered architecture has many advantages compared to the completely clustered structure, in terms of the complexity of the solution. Simulation results show that PSLL in the side clustered array can be reduced to more than -28 dB for an array of 40 elements. The PSLL was -17 dB in the case of a centrally clustered array, whereas the complexity percentage in the wholly clustered array method was 12.5%, while the same parameter for the partially clustered array method equaled 10%.

Keywords — beam pattern, clustered array, genetic algorithm, sidelobe.

## 1. Introduction

Synthesizing a large antenna array for modern communication applications, such as MIMO in 5G, is one of the most difficult challenges that designers face in practice. Therefore, it is necessary to use adapted techniques to simplify the construction process. Relying on subarrays is an important technique that is widely used in synthesizing large arrays to reduce complexity of the system by controlling the exciting points at the subarray level instead of the individual elements [1]. Such a reduction materially simplifies the manufacturing of large arrays and, thus, their cost may be reduced. The subarray technique means that the array containing N radiating elements is portioned into small classes (called clusters), with each cluster containing C radiating elements, so that  $C \ll N$ , and only C control points are required in the excitation of the array [2]. Each cluster has only one controller to feed the clusters in terms of amplitude and phase, or both. Although the desired beamwidth and directivity are still achieved, the use

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In the past years, several efficient methods have been proposed to address the problems of cluster synthesis. As far as we know, two types of methods based on evolutionary algorithms are used, such as genetic algorithm [2], [3] and particle swarm [5] algorithm. Mathematical analysis strategies, such as compressed sensing [6]-[8] are relied upon as well. In [2], two different structures are proposed to build linear clustered arrays based on a genetic algorithm. In the first structure, all the elements in the large array are fragmented into equalsized clusters, meaning that unwanted side lobes may appear in the radiation pattern. This problem was solved through the use of a second structure, where the edges of the linear array were divided into clusters, while the center elements remained without clustering, i.e. by optimizing the center elements individually. This leads to a lower reduction in the system's complexity. In [3], two structures based on the genetic algorithm are proposed as well. In the first structure, the radiating elements in the main array are divided into several irregular clusters, and in the second structure, only the elements located at the sides are divided into irregular clusters. In the two methods, high degrees of freedom were obtained to address the problem of undesirable side lobes, but the complexity remained rather high.

In this paper, the methods described in [2] and [3] have been extended and simplified by proposing fully and partially clustered architectures that are arranged in ascending order in terms of the number of elements in each cluster needed to build the desired radiation pattern under the applicable constraints. In the fully clustered array, the original linear array is divided into a group of clusters, with each one of them having an unequal number of elements. The clusters are arranged in an ascending order (in terms of the number of elements), starting from the edges and heading towards the center. Such a solution is referred to as a completely clustered array (CCA). Meanwhile, in the case of the partially clustered array, two



Fig. 1. CCA architecture.

architectures are proposed. In the first approach, the elements located on the sides are divided into ascending groupings, while the elements located in the middle are left without clustering and optimization. Such a solution is referred to as a side section clustered array (SSCA). In the second architecture, in turn, the elements located in the middle are divided into ascending groupings, while the elements located on the sides are left without clustering and optimization. Such a solution is referred to as a center section clustered array (CSCA). The sectionally clustered array architecture is characterized by a lower degree of complexity than the completely clustered array structure. The two proposed architectures are capable of ensuring good radiation properties with a simplified excitation network. To suppress unwanted quantized sidelobes existing in the desired radiation pattern, an upper boundary sidelobe limit constraint exempting the main lobe is proposed.

## 2. Formulation of the Proposed Clustered Structures

In this section, wholly and partially ascending clustered array architectures are presented and their practical advantages are demonstrated. For both architectures, the array factor and the cost function are declared with the constraint condition. The genetic algorithm is used under the following specifications: population size 60, mutation rate 0.16, and single-point crossover is used to construct the desired radiation pattern.

#### 2.1. CCA Elements

Let us consider a linear array containing an even number of N radiating elements. The elements are arranged and fed symmetrically around the center of the array, meaning that half of the elements are dealt with instead of all of them, as is the case in the optimization process. This means that

complexity will be reduced by half as well. In CCA, all the elements in the array are portioned into several C clusters and each c cluster contains B elements. B is always lower than N and N must be chosen so that the quotient of N/B is zero (i.e. C must be an integer). Figure 1 shows the structure of the proposed CCA. It can be seen that each cluster contains a set of elements. These clusters are arranged in an ascending order, starting from the edges to the center, depending on the number of elements. This means that the first cluster contains the least number of elements (for instance two) with the number of elements increasing by one in the next cluster, and so on. Such a gradual increase in the number of elements prevents the appearance of periodic quantized lobes in the desired radiation pattern.

To represent the mathematical function of controlling the upper bounds of the side lobes constraint in the optimization process, we start by writing the array factor (AF) of the clustered array as [3]:

$$AF(\theta) = 2\sum_{c=1}^{\frac{C}{2}} w_c \sum_{n=1}^{\frac{N}{2}} \beta_{cn} w_n \cos\left[\frac{2n-1}{2} k d \sin(\theta)\right], \quad (1)$$

$$\beta_{cn} = \begin{cases} 1, \text{ if the } n\text{-th elements belong to the } c\text{-th cluster} \\ 0, \text{ otherwise} \end{cases}, (2)$$

where  $w_c$  is the complex weight for each cluster output and equals  $w_c = A_c e^{jp_c}$ ,  $A_c$  and  $p_c$  are the amplitude and phase excitation, respectively,  $w_n$  is the complex weight of each independently exciting element in the main array and equals  $w_n = A_n e^{jp_n}$ ,  $A_n$  and  $p_c$  are the amplitude and phase excitation, respectively, k is the wave number equal to  $2\pi/\lambda$  and  $\lambda$  is the free space wavelength, d is the distance between any two elements in the linear array, while  $\theta$  is the angle of the main beam's direction.

It is clear from Eq. (1) that the number of clusters on each side of the linear array is equal and is arranged symmetrically.

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Fig. 2. SSCA architecture.



Fig. 3. CSCA architecture.

Thus, half of the clusters will be dealt with in the optimization process. To further simplify the exciting network, amplitudeonly weights are used, meaning  $p_c = p_n = 0$ . Therefore, Eq. (1) can be simplified to:

$$AF(\theta) = 2\sum_{c=1}^{\frac{C}{2}} A_c \sum_{n=1}^{\frac{N}{2}} \beta_{cn} A_n \cos\left[\frac{2n-1}{2} k d \sin(\theta)\right], \quad (3)$$

Equation (3) will be used in the optimization process to determine the optimal weight values for the elements of each cluster in the array system.  $A_c$  and  $A_n$  weights are optimized at the initial stage to construct the desired radiation pattern. After that, the average value of  $A_1, A_2, \ldots, A_c$  weights is taken within the cluster until C is calculated. In this way, the weights of  $A_1, A_2, \ldots, A_n$  are quantitatively determined by the value of  $A_c$ .

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In this section, a new method based on a partial cluster is presented. Two architectures are proposed to build a lowcomplexity array system. In the first architecture (SSCA), some elements in the linear array located on the edges are selected to be converted into ascendingly arranged clusters, with the center elements remaining without clustering and with uniform weights. Figure 2 shows this proposed architecture. The second architecture (CSCA) is the opposite of the first approach, as a group of elements close to the center is converted into ascending clusters, while the edge elements remain without clustering and improvement (i.e. uniform exciting – see Fig. 3).

The main advantage of these architectures is that the level of complexity of large array systems is lower compared to the





Fig. 4. Comparisons between the proposed CCA architecture and the existing structures from [2], [3].

first approach and to the structures proposed in [2], [3], while a good and desired beam pattern is guaranteed. Hence, the AF of these systems can be written as:

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$$AF(\theta)_{SSCA} = \underbrace{2\sum_{u=1}^{\frac{U}{2}} b_u \cos\left[\frac{2n-1}{2}kd\sin(\theta)\right]}_{\frac{U}{2} \text{ individually exciting elements}} + \underbrace{2\sum_{c=1}^{\frac{C}{2}} A_c \sum_{n=\frac{(N-U)+1}{2}} \beta_{cn}A_n \cos\left[\frac{2n-1}{2}kd\sin(\theta)\right]}_{\frac{C}{2} \text{ clusters on each side of the linear array}}$$
(4)

 $AF(\theta)_{CSCA} = 2\sum_{c=1}^{\frac{C}{2}} A_c \sum_{n=1}^{n=\frac{(N-U)+1}{2}} \beta_{cn} A_n \cos\left[\frac{2n-1}{2}kd\sin(\theta)\right]$  $\xrightarrow{\frac{C}{2} \text{ individually exciting elements}} + 2\sum_{n=1}^{\frac{U}{2}} h_n \cos\left[\frac{2n-1}{2}kd\sin(\theta)\right]$ (5)

$$+ \underbrace{2\sum_{u=1}^{\frac{\omega}{2}} b_u \cos\left[\frac{2n-1}{2} kd\sin(\theta)\right]}_{U}, \qquad (5)$$

 $\frac{U}{2}$  clusters individually exciting elements

where  $b_u$  represents the uniform weights of the individual elements in the main array. For the objective functions described in Eqs. (3)–(4) to work precisely in the optimization process, an additional condition is added to these functions that draw a definite borderline to control the levels of the side lobes in order to prevent them from being higher than the required levels. This will be referred to as the upper sidelobe boundary constraint (USBC). Then, it may be written as:

-0.6

 $Sin(\theta)$ 

-0.4

-0.2

0

-50

-1

-0.8

Additional cost function 
$$= \sum |AF(\theta) - USBC|^2$$
. (6)

## 3. Computer Simulation Results

In order to check the effectiveness of the proposed systems, i.e. CCA, SSCA, and CSCA, several tests have been performed. In all simulations, phase excitation ( $p_c = p_n = 0$ ) of the elements is set to zero, i.e. amplitude-only weights are used in the optimization process. The total number of elements used in the main array is chosen to be 40, with 20 elements on each side. The number of elements in each cluster *C* is specified, and then this number is increased by one in the next cluster. For example, if the number of ele-

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ments in the first cluster b = 2, then the second cluster will contain three elements b = 3, and so on (when b = 1, it represents a non-clustering condition). Because a symmetric linear array was used in all tests, only one aspect of the radiation pattern and amplitude distribution of the elements is shown.

In the first test, the wholly clustered array methods presented in [2], [3], i.e. the fully regular clustered elements (FRCE) method and the fully irregular clustered elements (FICE) method, are enhanced. Figure 4 shows the results of the proposed CCA method, compared with the results of FRCE and FICE methods in terms of radiation pattern and amplitude-only taper distribution. For the FRCE structure, the number of clusters of C = 5 on each side, and the number of elements in each cluster of c = 4. Here, we expect the appearance of uncontrolled periodic high-side lobes, and these lobes in-



Fig. 5. Comparisons between the proposed SSCA and CSCA architectures, and the existing structures from [2], [3].

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crease as the number of elements in the cluster grows. To solve this problem, the FICE method described in [3] is used. Here, the method depends on constructing irregular and unordered clusters, so there is a possibility of exceeding one of the periodic side lobes of the specified constraint border, especially if the lobes' constraint border is less than -25 dB. The proposed CCA approach solves the problem of periodic sidelobes definitively with an orderly arrangement of the complete feeding network, as such a solution facilitates its construction in practice.

In the second test, SSCA and CSCA architectures are illustrated. To show the performance of these two architectures in terms of obtaining the desired radiation pattern and simplifying the feed network significantly, these two methods are also compared with the partially regular clustered elements (PRCE) approach and the partially irregular clustered elements (PICE) scenario shown in papers [2] and [3], respectively. Through these four methods (PRCE, PICE, SSCA, and CSCA), we find that the two proposed architectures offer a significant reduction in the feeding network while reducing the level of side lobes (to a degree that is close to the one achieved by the other two methods). Figure 5 shows the results achieved by those four methods with 2N = 40, C = 4, and the number of individually excited elements b = 6. In PRCE and PICE methods, the individual elements are optimized, while in the two proposed methods, they are fed uniformly, without improvement. From Fig. 5 it may be seen that the level of the side lobes can be reduced to -28 dB in the three methods (PRCE, PICE, and SSCA), and to -17 dB in the case of the CSCA method, because the weights of the individual elements at the sides are higher than the weights of the clusters in the middle.

The complexity ratio of the three proposed methods (CCA, SSCA, and CSCA) may be defined as the ratio of the number of ascending clusters to the total number of elements in the main array:

Complexity of the CCA SSCA and CSCA methods  $= \frac{C}{2N} \times 100 \, [\%]$ . (7)



Fig. 6. Comparisons between the proposed SSCA and CSCA architectures, and the existing structures from [2], [3].



Fig. 7. Comparisons between the proposed SSCA and CSCA architectures, and the existing structures from [2], [3].

The complexity percentage in the CCA method was 12.5% (5 clusters were used symmetrically on each side), while in the SSCA and CSCA methods it equaled only 10%

(4 clusters were used symmetrically on each side with 6 uniform exciting elements), with the individual elements not taken into account in the complexity calculation. The computation of the complexity percentage in the PRCE and PICE methods will consider the individual elements, so the complexity percentage will increase. Its level amounted to 27.5%. It is noticed that there is a clear reduction in the complexity ratio in the proposed methods.

Figure 6 shows the variation of the cost function with the number of iterations for the proposed methods, comparing it with the methods described in [2] and [3]. It can be seen from this figure that the two methods (SSCA and CSCA) require the least number of iterations to converge, due to having the lowest number of degrees of freedom compared to other methods. This offers other advantages to the methods proposed in this paper.

The idea of an unequal ascending clustered array was extended to the two-dimensional (planar) array with a size of  $40 \times 40$ , and the results are illustrated in Fig. 7. The number of elements in each 2D cluster is selected to build two-dimensional ascending clusters. Here, the SSCA method was adopted when building the planar array. It is observed from Fig. 7 that  $11 \times 11$  elements in the center of the planar clustered array remain excited regularly, in addition to the presence of symmetry in the four quadrants of the amplitude distribution, which means that there is a significant reduction in complexity in practice. Moreover, radiation-related properties have been satisfied as well, as evidenced by the three-dimensional array pattern.

## 4. Conclusion

It is clear from the simulation tests that the desired beam pattern can be obtained with the required restrictions using three types of wholly and partially clustered architectures, depending on the ascending order of the clusters. The use of such architectures has resulted in a significant reduction of the exciting network and the associated costs. The complexity percentage in the proposed CCA method equaled 12.5%, amounting to 10% in the proposed CSCA and SS-CA methods. The use of an ascendingly arranged cluster architecture helps completely suppress the periodic side-lobe levels that were inevitable with regular and irregular clusters.

In addition, the suggested concept was expanded and deployed in 2D planar arrays, where each cluster's components may be created as tiles. Subsequently, the array radiation layout was adjusted by means of optimal sub-arraying arrangements. As a result, the suggested concept may be effectively implemented in large solutions, such as massive MIMO arrays.

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